ABSTRACT

KEYWORDS: Compact scheme; convection-diffusion; exponential scheme; higher order schemes; incompressible Navier-Stokes equations

An efficient and robust numerical scheme, in the finite difference framework, for solving linear and non-linear Convection-Diffusion Equations (CDE) has been developed. The standard numerical schemes generally perform poorly when there are sharp gradients in the solution caused by dominant convective coefficients. However, the extra evaluations of the source function and exponential fitting in the developed scheme have helped to achieve high accurate solutions even in the layered regions. It has been shown analytically that the coefficient matrices obtained from the developed scheme satisfy the conditions required to be an M-matrix that helped to solve non-linear and coupled non-linear convection-diffusion equations iteratively. The success of the iterative scheme led to formulation of a scheme for incompressible Navier-Stokes equations in its stream function and vorticity formulation.

Three constraints have been imposed on the scheme during the development, namely, the stencil to be compact, the coefficients of the scheme must contain exponential functions of convection-diffusion parameters and the step length, and finally, the scheme must be more than second order accurate. Due to the compact nature of the underlying stencil of the scheme, the resultant coefficient matrices of the scheme will have low band width (tri-diagonal for one-dimensional and block tri-diagonal for two-dimensional problems) and also no special care is required to approximate the governing equations at the nodes near the physical boundaries. The exponential coefficients of the scheme are effective in resolving the solutions without oscillations in the layered regions irrespective of their location. And finally, the higher order nature of the schemes improves the accuracy of the obtained solutions. These three important properties of the scheme are motivated to name the developed scheme as Exponential Compact Higher Order (ECHO) scheme. After success-
fully validating the developed scheme for linear, non-linear and coupled non-linear CDE, in both one and two-dimensions, it has been suitably modified to solve the incompressible Navier-Stokes equations. The validated scheme, finally, has been used to simulate the flow phenomenon in a lid-driven square cavity at high and very high Reynolds numbers.

To demonstrate the high resolution nature of the developed scheme, the characteristics of the one-dimensional scheme have been compared with the exact and existing schemes in the literature and demonstrated that the sixth order ECHO scheme is the best among the existing schemes. The concept of characteristic comparisons has also been extended for the schemes proposed for differential equations with variable coefficients as well as for two-dimensional equations. Using this analysis, the developed ECHO schemes have been categorized based on the number of free parameters in the scheme with respect to the order of the scheme. Finally, unsteady CDEs have been considered and an ECHO scheme for them has been developed with high spatial accuracy.