ABSTRACT

A complex network of coupled dynamical units has often been the object of active investigation in a wide range of interdisciplinary researches ranging from physical, biological to socio-technical and engineering systems. Diverse collective behaviors emanate in complex systems of interacting dynamical entities not only because of the local systems' unique attributes but mainly because of the variability in the fashion of interactions among the units. Both coupling topology and coupling function play quite decisive roles in the emergence of the collective asymptotic state. However, most of the existing investigations on coupled oscillators focus only on attractive pairwise interactions. Nevertheless, there exist various realistic, relevant scenarios in several scientific, technological, and natural instances, where competing types of couplings affect the collective behavior of coupled oscillator ensembles. One of the primary goals of this thesis is to study the diverse collective phenomena of an ensemble of coupled nonlinear oscillators under the coexistence of attractive and repulsive interactions in the static as well as in time-varying networks.

While complete synchronization is the least expected outcome in a static network with mixed positive-negative couplings, such an emergent state is still possible under the time-varying network formalisms. We encounter diverse collective behaviors ranging from complete and cluster synchronization to inhomogeneous small oscillation due to the interplay between the attractive and repulsive coupling strengths. In the absence of repulsive coupling, we successfully calculate the critical positive coupling strength required for acquiring complete synchronization using the time-average Laplacian matrix. We also recognize that suitable repulsive strength can make the error dynamics intermittent and leaves the signature of largely deviated extreme events. The formal practice to differentiate extreme events from other events is choosing a threshold $T = m + d\sigma$, where m is the sample mean of the observable and σ is the corresponding standard deviation. d is generally chosen ran-

domly from the interval $[4, 8]$. We provide an analytical formulation to derive a threshold $H_S = m + 8\sigma$ that distinguishes extreme events from other intermittent states. We further calculate an upper bound for the probability of occurrence of extreme events depending on the choice of H_S. Besides, we approximate the mean return interval of these extreme events using time series analogs of Kac's lemma.

In the case of static networks of coupled oscillators, we propose a unique algorithm by identifying a suitable repulsive path in a connected graph for establishing zero frustration (antiphase states). Moreover, we provide an analytical understanding with the help of graph theory to prove that antiphase synchronization is only possible in a connected network if it is bipartite in nature. In addition, we formulate a universal $0 - \pi$ algorithm for predicting the non-zero frustration value of arbitrary undirected non-bipartite graphs of attractive-repulsively coupled limit cycle oscillators. Using this algorithm, one can easily construct a sparse non-bipartite network with desired frustration from a highly diluted graph. We validate our findings in the single layer as well as multiplex networks. We identify a mixed state of interlayer antisynchronization and intralayer synchronization in a multiplex network when the interlinks' strength between layers is solely negative. We successfully calculate the amplitude of each Stuart-Landau oscillator, placed at each node of a multiplex network, exhibiting this mixed state. This derived attractor size remains independent of network size and agrees excellently with our numerical simulations for negative interlayer coupling strength. Apart from deriving the necessary condition for the existence of interlayer antisynchronization together with intralayer synchronization, we analytically derive the invariance of the intralayer synchronization manifold. We also derive the local stability condition of the interlayer antisynchronization state using the master stability function approach.