## Study of Poincaré-Hardy type inequalities and eigenvalue problems for second-order elliptic PDEs

The major text of this thesis is studying Poincaré-Hardy and Hardy-Rellich type inequalities on one of the most discussed Cartan-Hadamard manifold namely hyperbolic space and studying eigenvalue problems for second-order elliptic PDEs. The thesis is divided into two parts. In the first part we have centralized our attention on the following three problems:

• On some strong Poincaré inequalities on Riemannian models and their improvements.

• On higher order Poincaré inequalities with radial derivatives and Hardy improvements on the hyperbolic space.

• Hardy-Rellich and second order Poincaré identities on the hyperbolic space via Bessel pairs.

In the second part we have focused our essence on the following two problems:

• Generalized principal eigenvalues of convex nonlinear elliptic operators in  $R_{N}$ .

 $\cdot\,$  On ergodic control problem for viscous Hamilton-Jacobi equations for weakly coupled elliptic systems.