BIRKHOFF-JAMES ORTHOGONALITY AND DISTANCE FORMULAS IN C∗-ALGEBRAS AND

FOR TUPLES OF OPERATORS

This thesis is devoted to the study of Birkhoff-James orthogonality in C∗-algebras

and its applications in finding some distance formulas. We start with a thorough survey on the literature of Birkhoff-James orthogonality in the first chapter. Motivated

from the characterizations of Birkhoff-James orthogonality in the space of continuous

functions on a compact Haudorff space and the space of matrices, we prove a characterization of Birkhoff-James orthogonality in general C∗-algebras and Hilbert C∗-modules.

We provide three different proofs of our characterization with different themes in the

second chapter. It is shown that most of the results in the first chapter follow as applications of our characterization. Some new theorems are also proved. In the third chapter, applications of our characterization in the geometry of Banach spaces are mentioned and connection of orthogonality characterizations and distance problems is discussed. In the fourth chapter, we show the applications of our results for finding characterizations of orthogonality for tuples of operators. We also provide a distance formula in terms of variance of tuples of operators. Various possible directions for future work are mentioned throughout the thesis and are collected at the end for easy reference.