Preface

At the core of statistics lies the ideas of statistical inferences. Methods of statistical inference enable the investigator to argue from the observations in a particular sample to the general case; also help us to understand our world and make sound decisions about how to act. In general, inferences may be classified into two groups based on the logical reasoning used, namely inductive and deductive. In inductive inference, we are attempting to draw inferences from the particular to the general. Whereas deductive inference refers to arguments those are certain to be true by definition and derivation. Further, Inferences may also be classified into two groups based on the assumptions made about the model for the considered data set, namely parametric and non-parametric inferences. For more details about parametric and non-parametric inferences, readers may refer to Rohatgi and Saleh (2003), Walker and Lev (1953), Lehmann and Casella (1998), Boes et al. (1974), etc. Another classification of the statistical inference, depending upon the interpretation of probability and type of information used for inferences, that leads to two different paradigms for statistical inference; namely, the classical paradigm and the Bayesian paradigm. In Classical approach one uses the information available from the sample only and considers the parameters as unknown constants.

After classification of model based on information in hand, the next step is to estimate the parameters involved therein. In the statistical literature various classical and Bayesian estimation techniques are discussed for point estimation problem. These stated inferential procedures are based on data observed according to specific situations. Mostly, in the field of lifetime study, the data related to the units (man made or god made) are obtained from the lifetime experiments where randomly selected units are put on test under a specified condition. The lifetime of the units are observed. It may be noted that the observation thus obtained will naturally be obtained one after the other i.e. the life times are ordered observations in natural way. It may be noted here that the lifetime test as mentioned here will often be time consuming and costly. Thus one can not afford to wait till all the units put on test fails. Therefore many times, situations do arise when the units under study are lost or removed from the experiments while they are still alive i.e., we get censored data in such cases. If the point at which the experiment terminates is time dependent, it is called Type-I censoring. If it is unit dependent, it is called Type-II censoring. Depending on the need and practical considerations, various modified forms of censoring schemes have been discussed in the literature. Cohen (1965) developed a censoring scheme and named it progressive Type-II censoring scheme, which facilitates the removals of the units from the test at certain specified stages before termination of the experiment. For more details about various censoring scheme see Balakrishnan and Aggarwala (2000), Cohen and Norgaard (1977), Davis et al. (1979), Viveros and Balakrishnan (1994b), Balakrishnan and Sandhu (1995), etc.

Another situation which is often met in some of the life testing experiment is related to the limitation of the recording exact failure time of the units put on test. In such situations, random lifetime of interest is known only to lie within an interval instead of being observed exactly. This type of inspection plan give rise to a new censoring scheme, called interval censoring. Finkelstein (1986) has discussed many clinical trials and longitudinal studies falling in this category. According to Jianguo (2006), interval-censored failure time data occurs in many other areas also including demographic, epidemiological, financial, medical, sociological and engineering studies. Aggarwala (2001) proposed a combination of interval Type-I censoring and progressive censoring and called it as progressive interval Type-I (PITI) censoring which is having wide applications in clinical trials.

The present piece of work aims to explore the estimation method for PITI censoring scheme in Classical as well as in Bayesian paradigm for the considered distributions. It is also aimed to inquire whether censoring scheme have any significant effects on estimators and how much. We have dealt the Classical and Bayesian point estimation along with interval estimation of the parameters for considered life-time models under specified loss functions. For Bayesian computation, the MCMC techniques with M-H algorithm has been used to study the properties of the posterior distribution and proposed estimators. Under M-H algorithm, the asymptotic (normal) distribution of the posterior is considered as the proposal density for sample generation from that posterior.

The whole thesis is divided into 5 chapters. Among them, Chapter 1 provide a basic reading material so as to equip the reader for understanding the problems discussed in the following chapters. Although the topics considered have not been discussed in detail, it is attempted to provide sufficient references so that interested readers can look into the references to have the complete knowledge of the topics. Finally, we provided the motivation and summary of the present work.

The Chapter 2 deals with the various estimator for parameters of Generalised Exponential distribution under PITI censoring scheme when removals follow binomial distribution. We have checked the performances of the estimators under different considered removals and inspection plans. Since the ML estimators are not obtainable in explicit algebraic form, therefore, we propose to use some numerical iterative method, here we have used Newton-Raphson method in R-environment. Further, we observed that the Newton-Raphson method do not converge all the times, Therefore, we also used EM-Algorithm to obtain the ML estimates. Estimation of parameters under the Bayesian paradigm using non-informative and gamma priors when loss is considered to be SELF has also been considered. Bayes estimators also have not been obtained in explicit algebraic form. Therefore, we propose to use of M-H algorithm. Some diagnosis of M-H algorithm in the present problem is also discussed. We illustrate our proposed methodology with the two real examples. The first is related to the survival time of patients with Plasma Cell Myeloma data and the second is regarding to the number of revolutions (in million) before failure of groove ball bearings. On the basis of examples, we can't check the performance of estimators, so we conducted a Monte Carlo simulation study. For the simulation purpose, one need the generation of samples from specified model under considered censoring scheme. For this purpose we have developed an algorithm. Comparison of the performances of the various estimates of the parameters involved in the model along with reliability and hazard functions is made on the basis of their average absolute bias and mean square error (MSE). The content of this chapter is based on the author's paper "A. Kaushik, U. Singh and S. K. Singh (2017). *Estimations of the parameters of generalised exponential distribution under progressive interval Type-I censoring scheme with random removals*, *AUSTRIAN JOURNAL OF STATISTICS*."

In the previous chapter, we have discussed the procedure regarding the PITI censored sample considering the removals as Binomially distributed. In fact the probability of removal shall vary from patient to patient, also visit to visit and remains unknown to the experimenter. Thus, one should consider that the removal probability p to be random following a probability distribution. Keeping this point in mind, we propose beta-binomial distribution for the number of removals. In this chapter i.e. Chapter 3, we have discussed the problem of point and interval estimation of the shape and scale parameters of Weibull distribution based on PITI censoring with Beta-Binomial removals (PITICBBR). The Bayes estimators have been obtained by considering both symmetrical (SELF) and asymmetrical (GELF) loss functions. The estimators thus obtained, have been compared with the corresponding maximum likelihood estimators for their risks through Monte Carlo Simulation study. Further, expressions for the expected number of total failures (ENTF) have been derived for PITICBBR censoring scheme. Finally, a real data set has been considered to illustrate the practical utility of the paper. The content of this chapter is based on the author's paper "A. Kaushik, U. Singh and S. K. Singh (2015). *Bayesian Inference for the Parameters of Weibull Distribution under Progressive Type-I Interval Censored Data with Beta-binomial Removals*, *Communications in Statistics - Simulation and Computation*."

In the earlier chapters, we have seen that the inspection scheme and removals schemes do effect the performance of the estimators i.e. censoring scheme play a significant role to obtain the parameter's estimates. Abo-Eleneen (2011), Awad (2016) and Cho et al. (2015) compared the different censoring scheme in the experiments utilizing the entropy. The use of entropy measures for quantifying the information loss due to censoring has been proposed by Hollander et al. (1987). Kittaneh and Akbar (2014) discussed the efficiency of Type-I censored sample from Weibull distribution based on sub-measure of entropy. Kittaneh and El-Beltagy (2015) has compared censored sample utilizing the entropy. But all these studies are limited to Type-I, Type-II and progressive censoring. No work has been done in this direction for PITI censoring schemes. Even no one has attempted for the mathematical expression for Shannon entropy of PITI censored data. Motivated by the above facts, we are concerned here with the use of entropy for PITI censored data for comparing different censoring schemes. In Chapter 4, we have discussed Shannon entropy for the PITI censoring scheme. We have developed the methodology to obtained Shannon entropy for PITI censored sample. However, the entropy of PITI censored data is not so straightforward, because of it is having an m-dimensional summation. Here, we noticed that as m increases the time of computation increases with more faster rate. Hence, we suggest a simple and less time taking method using Monte Carlo technique to find entropy for considered situation. Finally, we apply our proposed methodology for computing the entropy for PITI censored samples for three distributions namely, Weibull, log-normal and GED.

As mentioned earlier, in many life-testing experiments, it is always more economical or practical to gauge observations PITI censored, than to record their actual measurements. In such situations, an important question is to determine the associated inspection times appropriately before conducting the experiment to assess the parameters of interest with least possible reduction in efficiency as compared to the exact observed situation. In this context, Lin et al. (2009) have developed some optimum inspection plan for log-normal distribution. They have used maximization of determinant of Fisher information matrix or minimization of determinant of variance-covariance matrix criterion for optimization. However, their study was concerned with log-normal distribution only. Therefore in Chapter 5, we propose a similar study for two parameters Rayleigh distribution with additional competitive optimality criterion for inspection times. For the above stated objective, first we derive the expected Fisher information matrix, generalised asymptotic variance, and asymptotic variance of population mean for a progressively interval Type-I censored sample. After that we describe some criteria for choosing inspection times for the optimal spaced inspection times. Further, we have propose two additional competitive optimality criterion for inspection times. For the illustration of our methodology, we perform a numerical study. The effect of the number of inspections and the choice of inspection times based on the asymptotic relative efficiencies (AREs) of the MLEs of the parameters are assessed. Finally, optimal PITI inspection plan have been obtained. ⊡